Problem 1.12

Calculate the average volume per molecule for an ideal gas at room temperature and atmospheric pressure. Then take the cube root to get an estimate of the average distance between molecules. How does this distance compare to the size of a small molecule like N_2 or H_2O?

Solution

An ideal gas obeys the ideal gas law.

$$PV = nRT$$

Solve for V/n, the average volume per mole.

$$\frac{V}{n} = \frac{RT}{P}$$

Then divide both sides by Avogadro's constant.

$$\frac{V}{nN_A} = \frac{RT}{PN_A}$$

The product of n and N_A is the number of molecules N.

$$\frac{V}{N} = \frac{RT}{PN_A}$$

V/N is the average volume per molecule, the desired quantity. Room temperature is 25°C, or 298.15 K, and atmospheric pressure is 1 atm, or 101 325 Pa.

$$\frac{V}{N} = \frac{\left(8.314\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(298.15\text{ K})}{(101\,325\text{ Pa})\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)} \approx 4.06\times10^{-26}\ \frac{\text{m}^3}{\text{molecule}}$$

If each molecule in the gas is evenly spaced apart and put at the center of a cube (as in a lattice), this number V/N tells how much space each cube takes up. Taking the cube root of this number gives the length of a cube's edge, which is the average intermolecular distance.

$$\sqrt[3]{\frac{V}{N}} \approx 3.44\times10^{-9}\text{ m} = 3.44\text{ nm}$$

Small molecules, such as N_2 or H_2O, are on the order of angstroms (10^{-10} m) in size. The intermolecular distance is therefore about ten times the size of small molecules.